Short Course on Experimental Dynamic Substructuring

Module #7: Decoupling Techniques

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Short Course Notes For:
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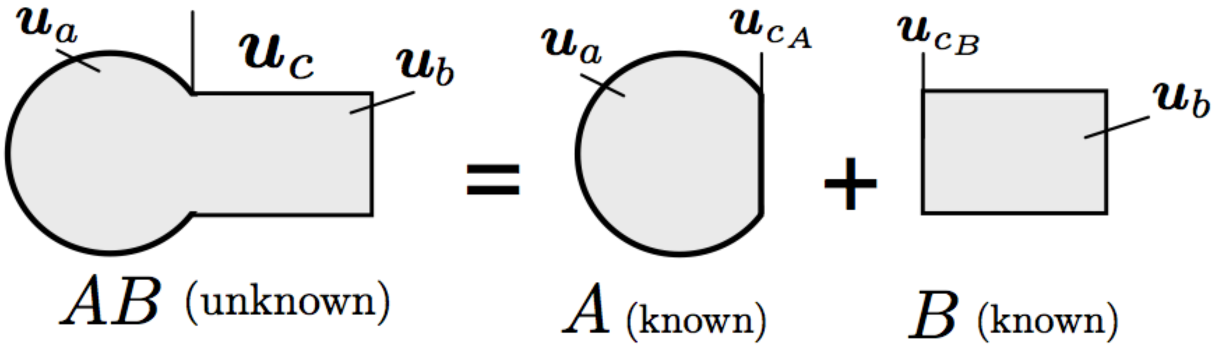
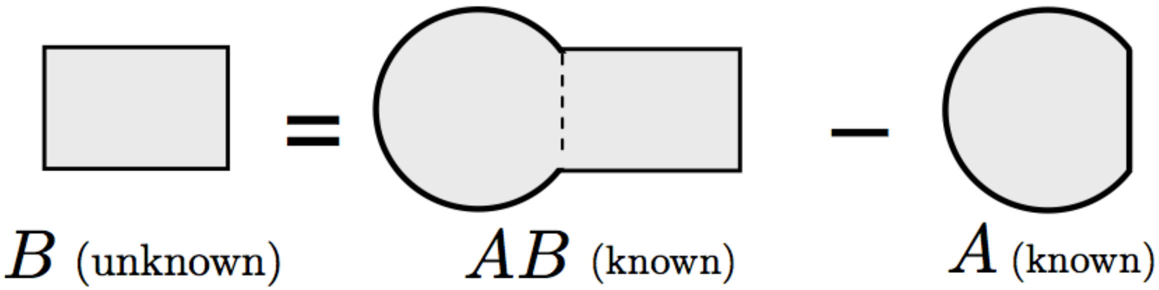
What is decoupling ?

- The basic idea:

Substructure coupling:



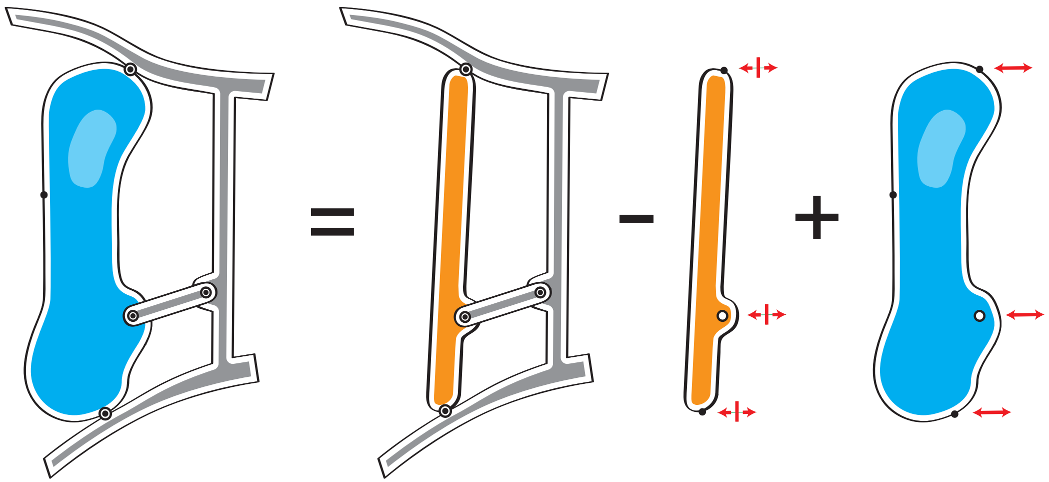
Substructure decoupling:



Why decoupling ?

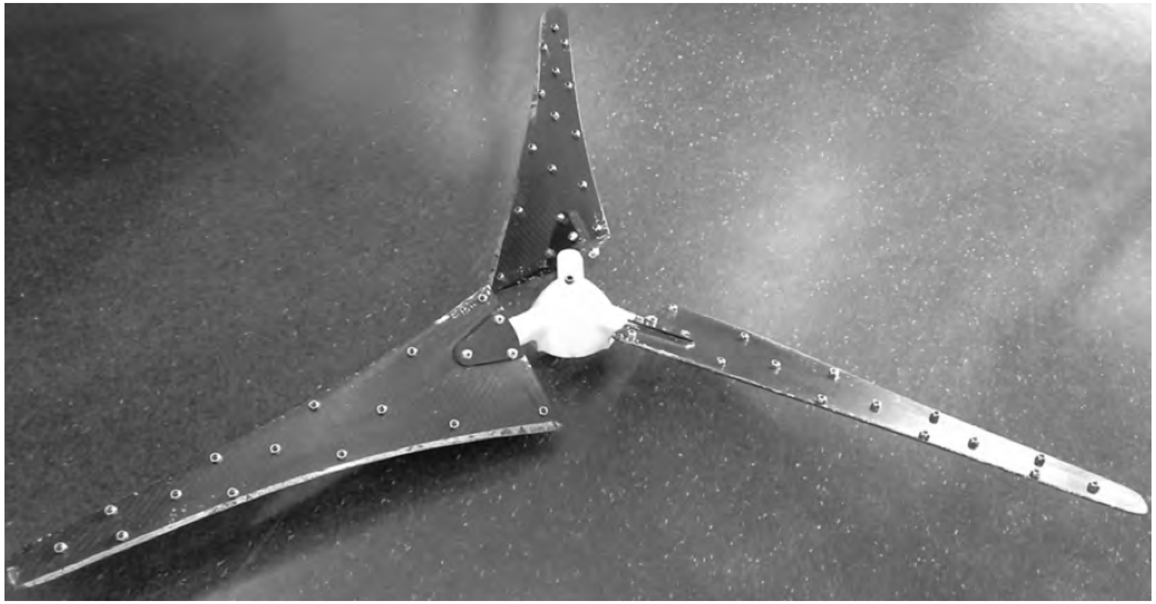
- Find the FRFs of components that can not well measured when standalone (e.g. removing dynamics of test-frame)

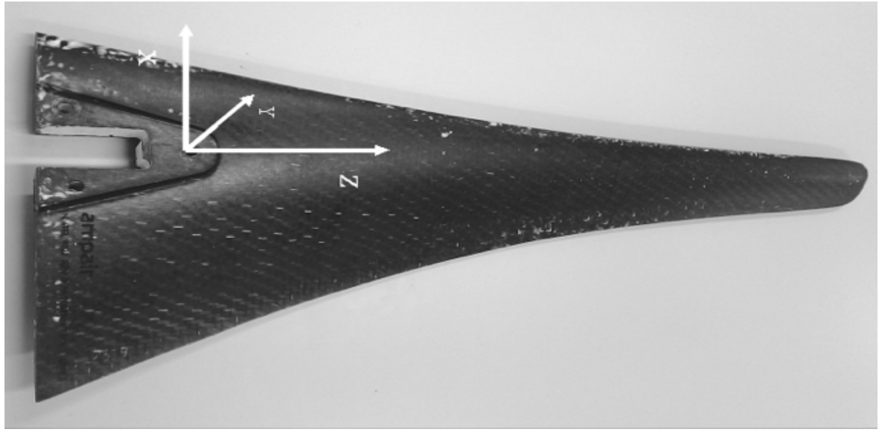
- Obtain the FRFs of a component including its interface dynamics (damping and stiffness). Example
  - steering gear [10],
  - blades attached to hub (Ampair wind Turbine benchmark [11])

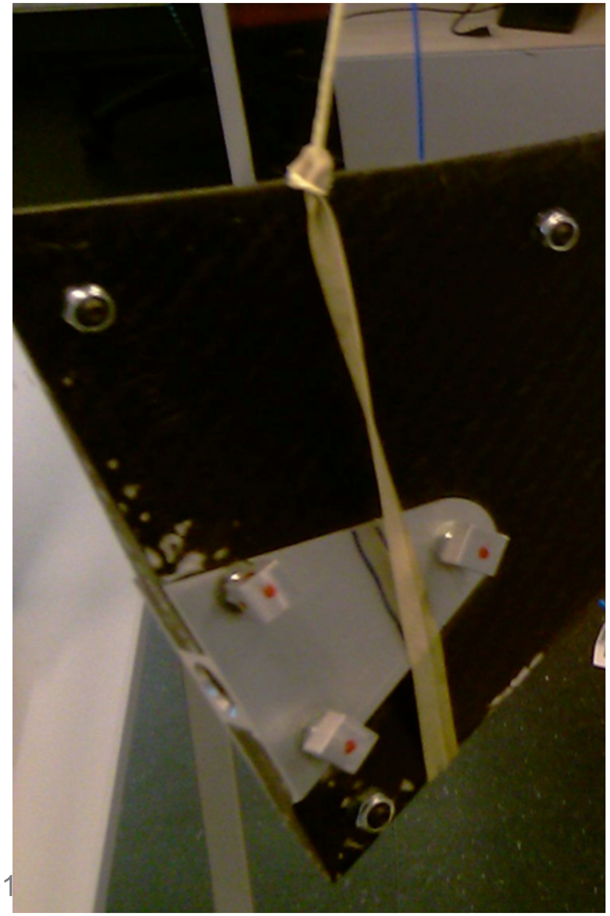


Why decoupling ?

- continued …
  - blades attached to hub (Ampair wind Turbine benchmark [11])



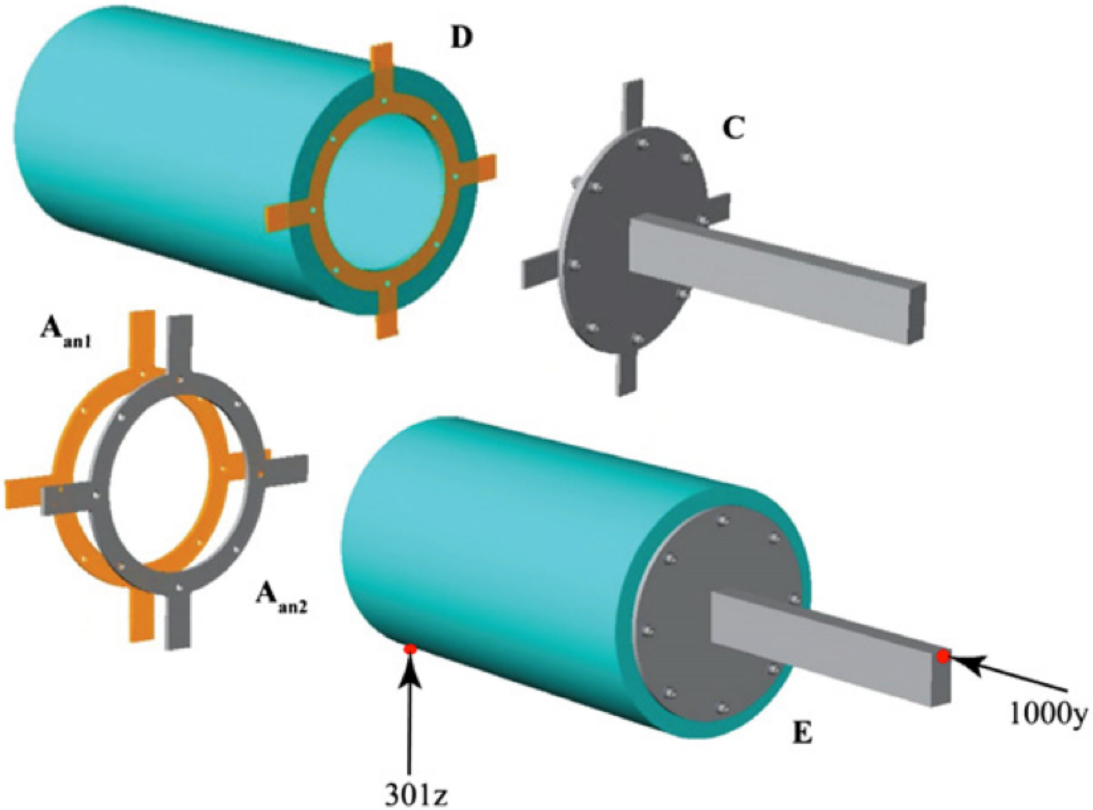




Why decoupling ?

- Let the component vibrate during tests in “modes” closer to the behavior of the component in the full assembly (the “inversion” in the FRF has ”less work to do”): idea of transmission simulator [7]

A+B = (A+C)+(B+C) -2 C



see MOD08

Why decoupling ?

But if coupling of substructures with FBS is tricky, isn't it even more tricky to apply decoupling ?

- … since in decoupling we have redundant information one cane mitigate measurement errors.
- The basic theory is very similar to the FBS for coupling (MOD02), but additional extensions can be devised

These notes are mainly based on [1].

Note:
A similar but different approach is called "inverse Substructuring" [2]. Whereas here we assume one of the substructure known. Inverse Substructuring aims at deriving the FRFs of both components (none of them being known at forehand). Requires measuring the response at both sides of a flexible interface between components.

Outline

- **Recap of MOD02:**
  Frequency Based Substructuring (FBS)3-field form
- **Frequency Based Substructuring: Basics**
- **Frequency Based Substructuring: A big Family**
  - Method 1 – Standard Decoupling
  - Method 2 – Extended Interface Decoupling
  - Method 4 – Non-Collocated Overdetermined
  - Method 5 – Weak, Interface Filtering
  - Method 6 – Weak, A Filtering
  - Summary of Variants
- **Numerical Example / Experimental Example**

References and bibliography

Recap of MOD02: Frequency Based Substructuring (FBS)

$\boldsymbol{Z}$: block-diagonal of substructure impedances (free interface)

$\boldsymbol{B}$: Boolean matrix (-/+ 1) describing connectivity of dofs on interface

λ : interface forces

$$\begin{bmatrix} \boldsymbol{Z} & \boldsymbol{B}^T \\ \boldsymbol{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \lambda \end{bmatrix} = \begin{bmatrix} \boldsymbol{f} \\ \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{u} = \left(\boldsymbol{Y} - \boldsymbol{Y}\boldsymbol{B}^T\left(\boldsymbol{B}\boldsymbol{Y}\boldsymbol{B}^T\right)^{-1}\boldsymbol{B}\boldsymbol{Y}\right)\boldsymbol{f}$$

incompatibility between substructures when no connection

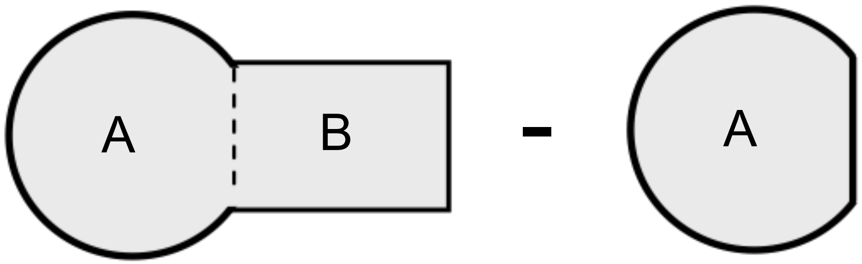
interface force λ needed for compatibility

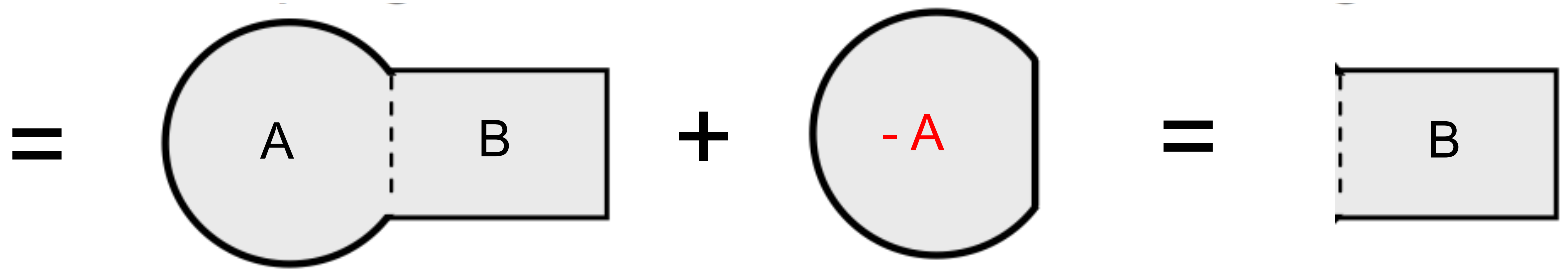
response to applied forces

response to interface forces

Frequency Based Decoupling: Basic

If we want to “subtract” two substructures, we can just do





$$\begin{bmatrix} \boldsymbol{Z}^{AB} & \boldsymbol{0} & \boldsymbol{B}^{AB^T} \\ \boldsymbol{0} & -\boldsymbol{Z}^{A} & \boldsymbol{B}^{A^T} \\ \boldsymbol{B}^{AB} & \boldsymbol{B}^{A} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{AB} \\ \boldsymbol{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^{AB} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

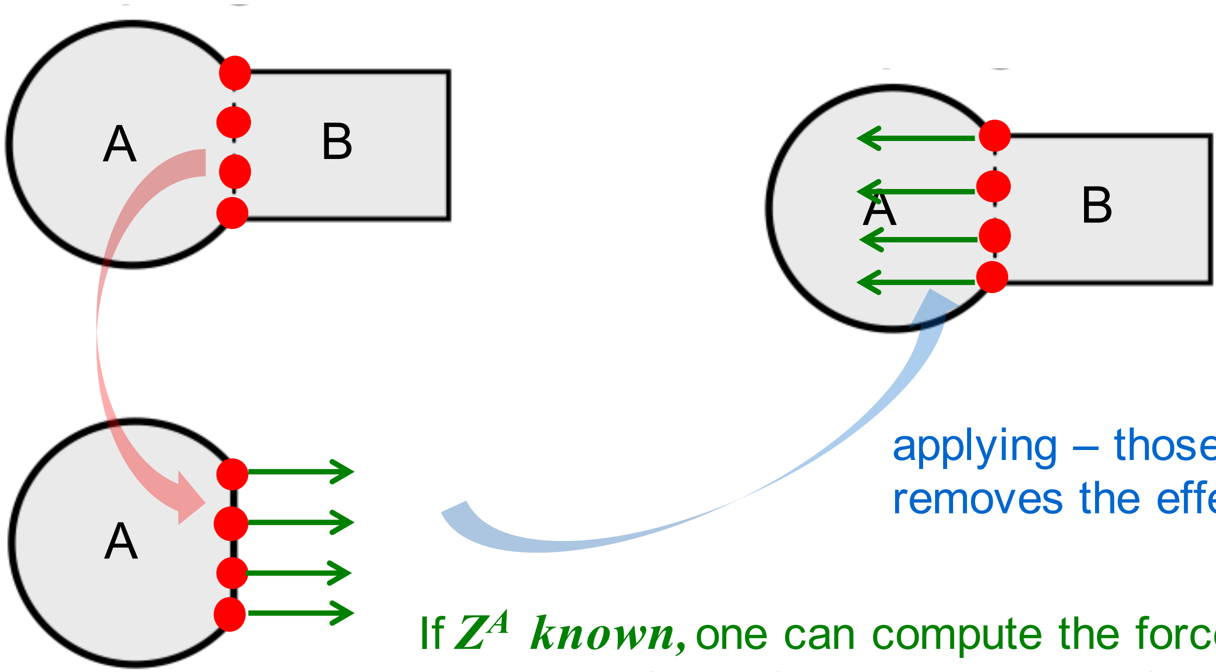
Allows computing the dynamics of B when the FRFs of AB and A are known: Substructuring Decoupling

Frequency Based Decoupling: Basics

Another interpretation:

$$\begin{bmatrix} \mathbf{Z}^{AB} & \mathbf{0} & \mathbf{B}^{AB^T} \\ \mathbf{0} & -\mathbf{Z}^{A} & \mathbf{B}^{A^T} \\ \mathbf{B}^{AB} & \mathbf{B}^{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{AB} \\ \mathbf{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{AB} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \xleftrightarrow{\text{eq. x (-1)}} \begin{bmatrix} \mathbf{Z}^{AB} & \mathbf{0} & \mathbf{B}^{AB^T} \\ \mathbf{0} & \mathbf{Z}^{A} & -\mathbf{B}^{A^T} \\ \mathbf{B}^{AB} & \mathbf{B}^{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{AB} \\ \mathbf{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{AB} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

Response of the system AB as if A would be absent:



applying – those forces
removes the effect of the presence of A

If $\mathbf{Z}^A$ ***known,*** one can compute the forces needed to
let the interface of A move as in the full system AB

Frequency Based Decoupling: Basics

Working it out …

$$\begin{bmatrix} \boldsymbol{Z}^{AB} & \mathbf{0} & \boldsymbol{B}^{AB^T} \\ \mathbf{0} & -\boldsymbol{Z}^{A} & \boldsymbol{B}^{A^T} \\ \boldsymbol{B}^{AB} & \boldsymbol{B}^{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{AB} \\ \boldsymbol{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^{AB} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \Rightarrow \begin{array}{l} \boldsymbol{u}^{AB} = \mathbf{Y}^{AB}\left(\boldsymbol{f}^{AB} - \boldsymbol{B}^{AB^T}\lambda\right) \\ \boldsymbol{u}^{A} = -\mathbf{Y}^{A}\left(-\boldsymbol{B}^{A^T}\lambda\right) \end{array}$$

$$\left[\boldsymbol{B}^{AB}\mathbf{Y}^{AB}\boldsymbol{B}^{AB^T} - \boldsymbol{B}^{A}\mathbf{Y}^{A}\boldsymbol{B}^{A^T}\right]\lambda = \boldsymbol{B}^{AB}\mathbf{Y}^{AB}\boldsymbol{f}^{AB}$$

$$\lambda = \left[\boldsymbol{B}^{AB}\mathbf{Y}^{AB}\boldsymbol{B}^{AB^T} - \boldsymbol{B}^{A}\mathbf{Y}^{A}\boldsymbol{B}^{A^T}\right]^{-1}\boldsymbol{B}^{AB}\mathbf{Y}^{AB}\boldsymbol{f}^{AB}$$

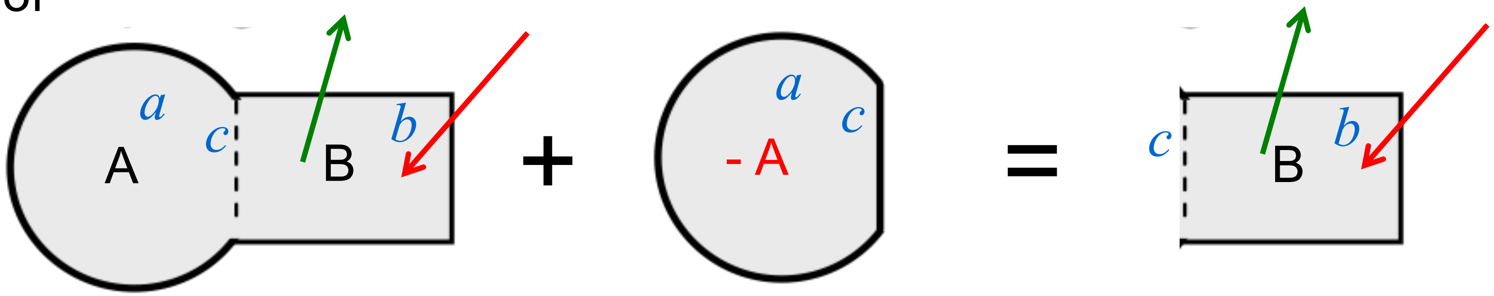
$$\boldsymbol{u}^{AB} = \left(\mathbf{Y}^{AB} - \mathbf{Y}^{AB}\boldsymbol{B}^{AB^T}\left[\boldsymbol{B}^{AB}\mathbf{Y}^{AB}\boldsymbol{B}^{AB^T} - \boldsymbol{B}^{A}\mathbf{Y}^{A}\boldsymbol{B}^{A^T}\right]^{-1}\boldsymbol{B}^{AB}\mathbf{Y}^{AB}\right)\boldsymbol{f}^{AB}$$

$$\boldsymbol{u}^{A} = \mathbf{Y}^{A}\boldsymbol{B}^{A^T}\left[\boldsymbol{B}^{AB}\mathbf{Y}^{AB}\boldsymbol{B}^{AB^T} - \boldsymbol{B}^{A}\mathbf{Y}^{A}\boldsymbol{B}^{A^T}\right]^{-1}\boldsymbol{B}^{AB}\mathbf{Y}^{AB}\boldsymbol{f}^{AB}$$

pffff ... but in here is the FRF of B when isolated (see next page)

Frequency Based Decoupling: Basics

Working it out for



Assuming (without loss of generality) the numbering of dofs such that

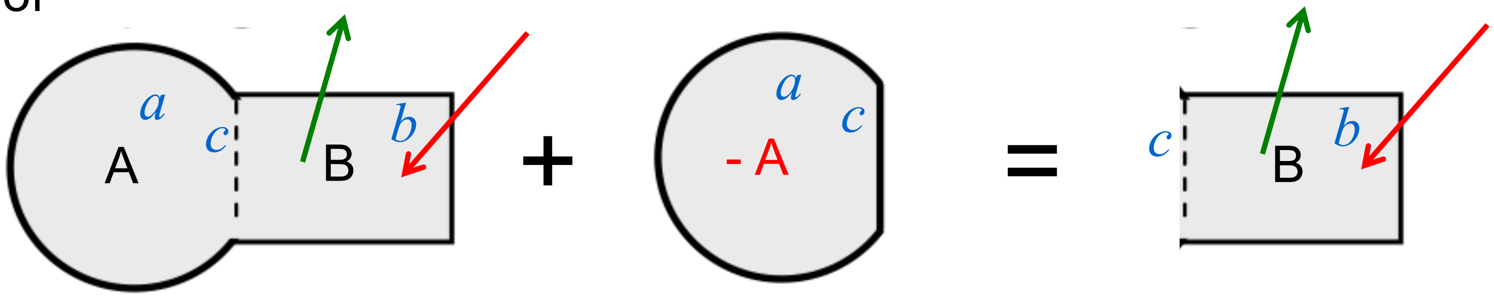
$$\boldsymbol{Y}^{AB} = \begin{bmatrix} \mathbf{Y}_{aa}^{AB} & \mathbf{Y}_{ac}^{AB} & \mathbf{Y}_{ab}^{AB} \\ \mathbf{Y}_{ca}^{AB} & \mathbf{Y}_{cc}^{AB} & \mathbf{Y}_{cb}^{AB} \\ \mathbf{Y}_{ba}^{AB} & \mathbf{Y}_{bc}^{AB} & \mathbf{Y}_{bb}^{AB} \end{bmatrix} \qquad \boldsymbol{u}^{AB} = \begin{bmatrix} \boldsymbol{u}_a^{AB} \\ \boldsymbol{u}_c^{AB} \\ \boldsymbol{u}_b^{AB} \end{bmatrix} \qquad \boldsymbol{f}^{AB} = \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{f}_b^{AB} \end{bmatrix}$$

$$\boldsymbol{Y}^{A} = \begin{bmatrix} \mathbf{Y}_{aa}^{A} & \mathbf{Y}_{ac}^{AB} \\ \mathbf{Y}_{ca}^{A} & \mathbf{Y}_{cc}^{A} \end{bmatrix} \qquad \boldsymbol{u}^{A} = \begin{bmatrix} \boldsymbol{u}_a^{A} \\ \boldsymbol{u}_c^{A} \end{bmatrix}$$

$$\boldsymbol{B} = [\boldsymbol{B}^{AB} \;\; \boldsymbol{B}^{A}] = [\mathbf{0} \;\; \boldsymbol{I} \;\; \mathbf{0} \;\; | \;\; \mathbf{0} \;\; -\boldsymbol{I}]$$

Frequency Based Decoupling: Basics

Working it out for



One finds ….

$$\begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c \\ \boldsymbol{u}_b \end{bmatrix}^{AB} = \left(\begin{bmatrix} \mathbf{Y}_{ab}^{AB} \\ \mathbf{Y}_{cb}^{AB} \\ \mathbf{Y}_{bb}^{AB} \end{bmatrix} - \begin{bmatrix} \mathbf{Y}_{ac}^{AB} \\ \mathbf{Y}_{cc}^{AB} \\ \mathbf{Y}_{bc}^{AB} \end{bmatrix} \left[\mathbf{Y}_{cc}^{AB} - \mathbf{Y}_{cc}^{A} \right]^{-1} \mathbf{Y}_{cb}^{AB} \right) \boldsymbol{f}_b^{AB}$$

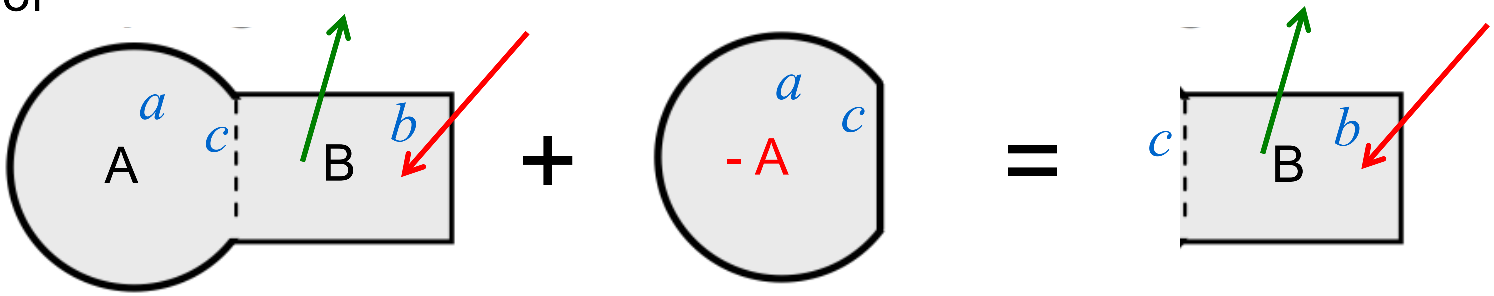
useful

$$\begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c \end{bmatrix}^{A} = - \begin{bmatrix} \mathbf{Y}_{ac}^{A} \\ \mathbf{Y}_{cc}^{A} \end{bmatrix} \left[\mathbf{Y}_{cc}^{AB} - \mathbf{Y}_{cc}^{A} \right]^{-1} \mathbf{Y}_{cb}^{AB} \boldsymbol{f}_b^{AB}$$

$$\boxed{\boldsymbol{u}_b = \left(\mathbf{Y}_{bb}^{AB} - \mathbf{Y}_{bc}^{AB} \left[\mathbf{Y}_{cc}^{AB} - \mathbf{Y}_{cc}^{A} \right]^{-1} \mathbf{Y}_{cb}^{AB} \right) \boldsymbol{f}_b^{AB}}$$

Frequency Based Decoupling: Basics

Working it out for



If errors in measurements of AB and/or A, might be useful to impose this compatibiiity explicitly

$$\begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c \\ \boldsymbol{u}_b \end{bmatrix}^{AB} = \left(\begin{bmatrix} \mathbf{Y}_{ab}^{AB} \\ \mathbf{Y}_{cb}^{AB} \\ \mathbf{Y}_{bb}^{AB} \end{bmatrix} - \begin{bmatrix} \mathbf{Y}_{ac}^{AB} \\ \mathbf{Y}_{cc}^{AB} \\ \mathbf{Y}_{bc}^{AB} \end{bmatrix} \left[\mathbf{Y}_{cc}^{AB} - \mathbf{Y}_{cc}^{A} \right]^{-1} \mathbf{Y}_{cb}^{AB} \right) \boldsymbol{f}_b^{AB}$$

identical because „A in AB“ is equal to „A subtracted“

identical by construction

$$\begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c \end{bmatrix}^{A} = - \begin{bmatrix} \mathbf{Y}_{ac}^{A} \\ \mathbf{Y}_{cc}^{A} \end{bmatrix} \left[\mathbf{Y}_{cc}^{AB} - \mathbf{Y}_{cc}^{A} \right]^{-1} \mathbf{Y}_{cb}^{AB} \boldsymbol{f}_b^{AB}$$

$$\boxed{\boldsymbol{u}_b = \left(\mathbf{Y}_{bb}^{AB} - \mathbf{Y}_{bc}^{AB} \left[\mathbf{Y}_{cc}^{AB} - \mathbf{Y}_{cc}^{A} \right]^{-1} \mathbf{Y}_{cb}^{AB} \right) \boldsymbol{f}_b^{AB}}$$

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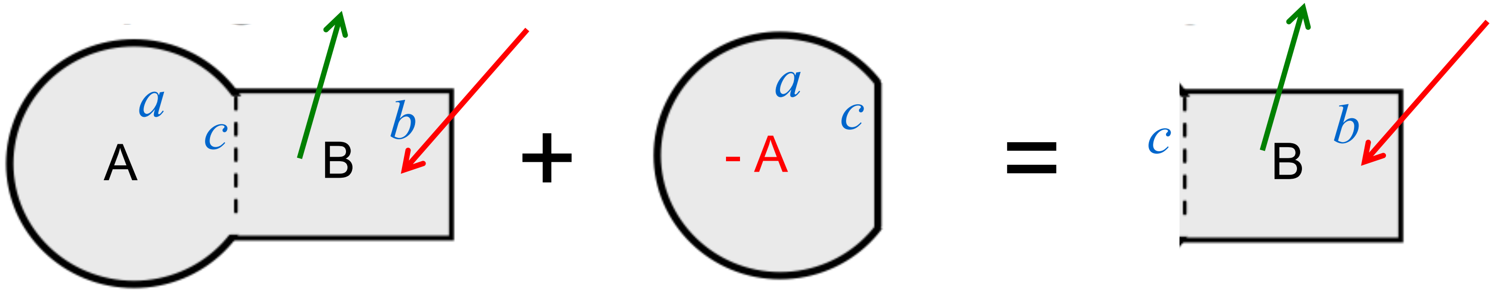
Frequency Based Decoupling: A Big Family

To understand how various decoupling method can be devised, let us reconsider



$$\begin{bmatrix} \boldsymbol{Z}^{AB} & \boldsymbol{0} & \boldsymbol{B}^{AB^T} \\ \boldsymbol{0} & -\boldsymbol{Z}^{A} & \boldsymbol{B}^{A^T} \\ \boldsymbol{B}^{AB} & \boldsymbol{B}^{A} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{AB} \\ \boldsymbol{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^{AB} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

- writing explicitly that AB is an assembly of A and B
- noting that "A in AB" can be different from A alone due to measurement/modeling errors

$$\tilde{\boldsymbol{Z}}^{A} = \boldsymbol{Z}^{A} + \boldsymbol{\Delta}$$

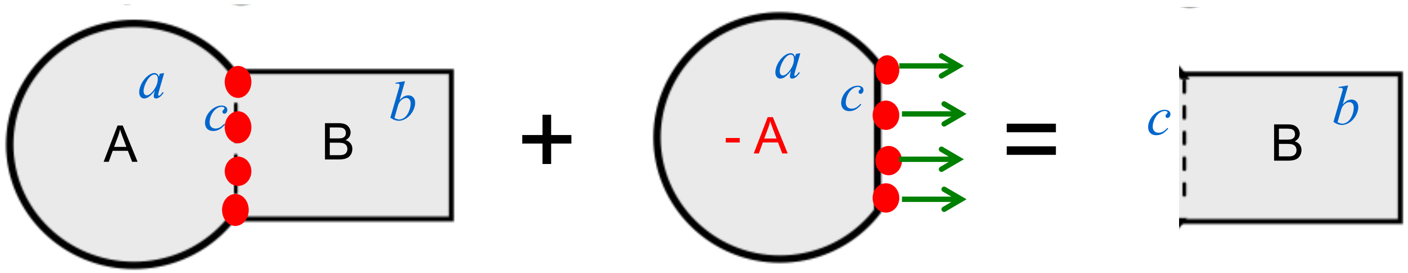
$$\left[\begin{array}{ccc|cc} \boldsymbol{Z}^{A} & \boldsymbol{0} & \boldsymbol{B}^{A^T} & \boldsymbol{0} & -\boldsymbol{E}^{A^T} \\ \boldsymbol{0} & \boldsymbol{Z}^{B} & \boldsymbol{B}^{B^T} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{B}^{A} & \boldsymbol{B}^{B} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \hline \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\tilde{\boldsymbol{Z}}^{A} & \boldsymbol{E}^{A^T} \\ -\boldsymbol{C}^{A} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{C}^{A} & \boldsymbol{0} \end{array}\right] \left[\begin{array}{c} \boldsymbol{u}_A^{AB} \\ \boldsymbol{u}_B^{AB} \\ \boldsymbol{\lambda}^{AB} \\ \hline \boldsymbol{u}^{A} \\ \boldsymbol{\lambda} \end{array}\right] = \left[\begin{array}{c} \boldsymbol{0} \\ \boldsymbol{f}_B^{AB} \\ \boldsymbol{0} \\ \hline \boldsymbol{0} \\ \boldsymbol{0} \end{array}\right] \Leftrightarrow \boxed{\begin{bmatrix} \boldsymbol{Z}^{AB} & \boldsymbol{0} & \boldsymbol{E}^{AB^T} \\ \boldsymbol{0} & -\tilde{\boldsymbol{Z}}^{A} & \boldsymbol{E}^{A^T} \\ \boldsymbol{C}^{AB} & \boldsymbol{C}^{A} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{AB} \\ \boldsymbol{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^{AB} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}}$$

to be determined so that the result is the free FRF of B

If $\boldsymbol{E}=\boldsymbol{C}$ then "collocated"
If not, "non-collocated"

Frequency Based Decoupling: A Big Family

Method 1: ***standard decoupling*** (collocated on interface)



note: forces drawn only in standalone A for simplicity

$$\begin{bmatrix} \boldsymbol{Z}^{AB} & \boldsymbol{0} & \boldsymbol{E}^{AB^T} \\ \boldsymbol{0} & -\tilde{\boldsymbol{Z}}^{A} & \boldsymbol{E}^{A^T} \\ \boldsymbol{C}^{AB} & \boldsymbol{C}^{A} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{AB} \\ \boldsymbol{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^{AB} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{C}^{AB} = \boldsymbol{E}^{AB} = [\boldsymbol{0}_{ca} \;\; \boldsymbol{I}_{cc} \;\; \boldsymbol{0}_{cb}]$$

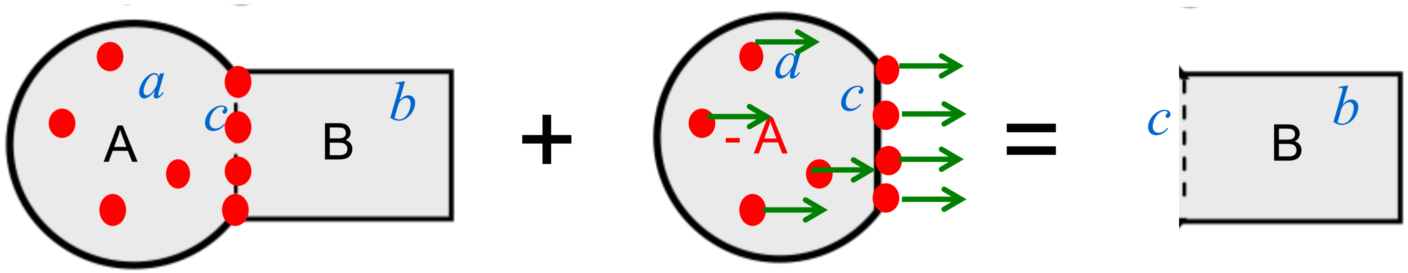
corresponds to the method derived in the previous slides:

$$\boldsymbol{C}^{A} = \boldsymbol{E}^{A} = [\boldsymbol{0}_{ca} \;\; -\boldsymbol{I}_{cc}]$$

 simple but very sensitive to errors !

Frequency Based Decoupling: A Big Family

Method 2: "***extended interface" decoupling*** (collocated on interface and in A)



$$\begin{bmatrix} \boldsymbol{Z}^{AB} & \boldsymbol{0} & \boldsymbol{E}^{AB^T} \\ \boldsymbol{0} & -\tilde{\boldsymbol{Z}}^{A} & \boldsymbol{E}^{A^T} \\ \boldsymbol{C}^{AB} & \boldsymbol{C}^{A} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{AB} \\ \boldsymbol{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^{AB} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{C}^{AB} = \boldsymbol{E}^{AB} = \begin{bmatrix} \boldsymbol{I}_{aa} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{cc} & \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{C}^{A} = \boldsymbol{E}^{A} = \begin{bmatrix} -\boldsymbol{I}_{aa} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{I}_{cc} \end{bmatrix}$$

Request compatibility on interface and in some dofs of A

🙂 . uses more info of the problem

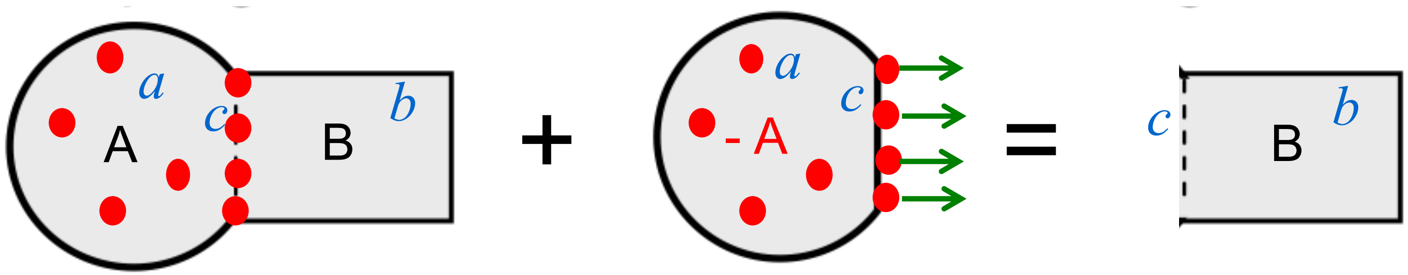
🙁 . still strong compatibility so sensitive to errors
. interface problem badly condition (could use SVD)

[3,4]

Frequency Based Decoupling: A Big Family

Method 3: ***Non-Collocated (NC) overdetermined***

forces on interface / compatibility on interface & in A



$$\begin{bmatrix} \boldsymbol{Z}^{AB} & \boldsymbol{0} & \boldsymbol{E}^{AB^T} \\ \boldsymbol{0} & -\tilde{\boldsymbol{Z}}^{A} & \boldsymbol{E}^{A^T} \\ \boldsymbol{C}^{AB} & \boldsymbol{C}^{A} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{AB} \\ \boldsymbol{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^{AB} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{E}^{AB} = [\boldsymbol{0}_{ca} \;\; \boldsymbol{I}_{cc} \;\; \boldsymbol{0}_{cb}]$$

$$\boldsymbol{E}^{A} = [\boldsymbol{0}_{ca} \;\; -\boldsymbol{I}_{cc}]$$

$$\boldsymbol{C}^{AB} = \begin{bmatrix} \boldsymbol{I}_{aa} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{cc} & \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{C}^{A} = \begin{bmatrix} -\boldsymbol{I}_{aa} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{I}_{cc} \end{bmatrix}$$

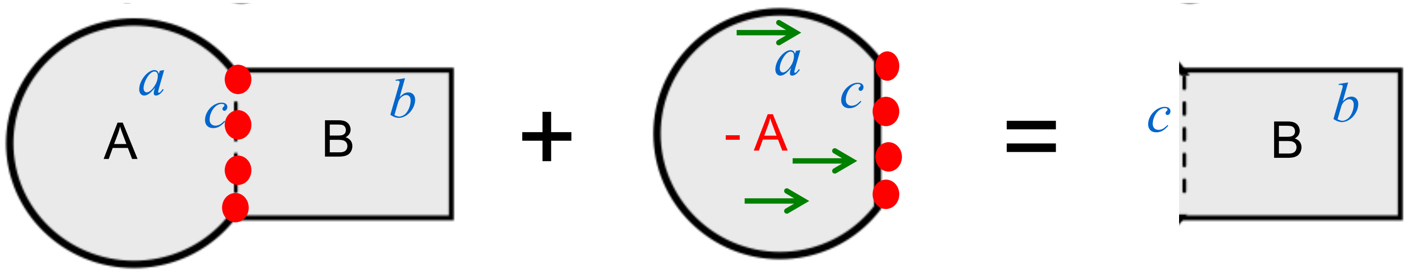
More compatibility asked for than forces :

. uses more info of the problem
. only approximate (weak compatibility)
. interface problem overdetermined (least square)

[5,6]

Frequency Based Decoupling: A Big Family

Method 4: ***Non-Collocated (NC) internal***
forces in A / compatibility on interface



$$\begin{bmatrix} \boldsymbol{Z}^{AB} & \boldsymbol{0} & \boldsymbol{E}^{AB^T} \\ \boldsymbol{0} & -\tilde{\boldsymbol{Z}}^{A} & \boldsymbol{E}^{A^T} \\ \boldsymbol{C}^{AB} & \boldsymbol{C}^{A} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{AB} \\ \boldsymbol{u}^{A} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^{AB} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{E}^{AB} = [\boldsymbol{I}_{aa} \;\; \boldsymbol{0}_{ac} \;\; \boldsymbol{0}_{ab}]$$
$$\boldsymbol{E}^{A} = [-\boldsymbol{I}_{aa} \;\; \boldsymbol{0}_{ac}]$$

$$\boldsymbol{C}^{AB} = [\boldsymbol{0} \;\; \boldsymbol{I}_{cc} \;\; \boldsymbol{0}]$$
$$\boldsymbol{C}^{A} = [\boldsymbol{0} \;\; -\boldsymbol{I}_{cc}]$$

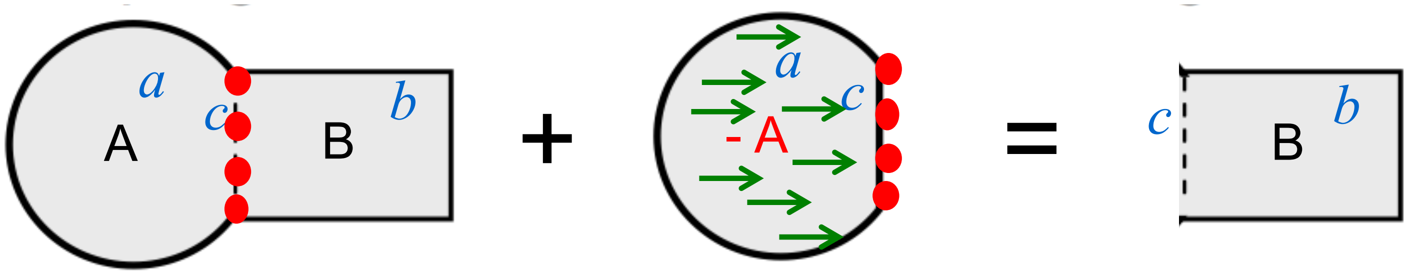
Works if the interface dofs are controllable from the dofs inside A

. no need to have driving points at interface
(can be difficult to measure)
. approximate (weak) compatibility
. interface problem overdetermined if more c-dofs than a-dofs

Frequency Based Decoupling: A Big Family

Many other variants can be imagined by choosing other locations for the compatibility conditions and for the forces.

One could even imagine taking more forces than compatibility conditions. In that case the interface problem would be underdetermined and a least square procedure would choose the force distribution with minimum norm.



This would correspond to a weakening on the force side.

The next variants introduce weakening explicitly in the standard formulas, similar to what was done in the coupling of substructures through FBS (MOD02)

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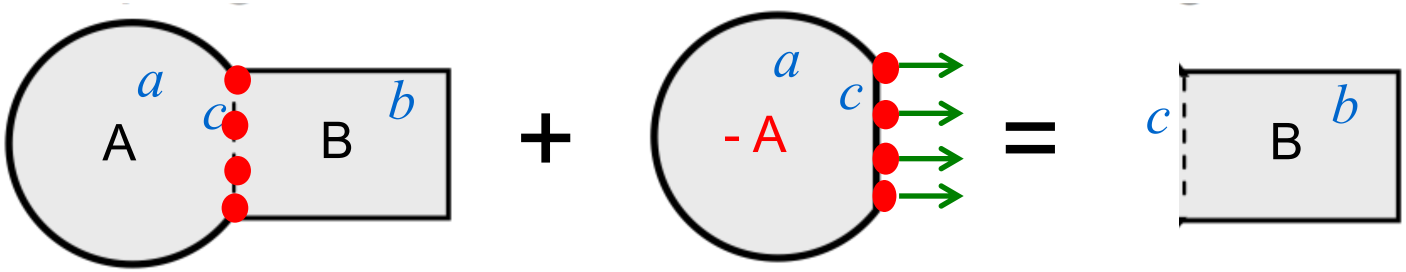
Frequency Based Decoupling: A Big Family

When measurements noisy, interface very stiff, or when coupling modal models (risk of locking), it is necessary to relax the compatibility requirements between AB and A.

Here we will only consider weakening through filtering (see also MOD02).

Starting again from the standard method (collocated on inteface)

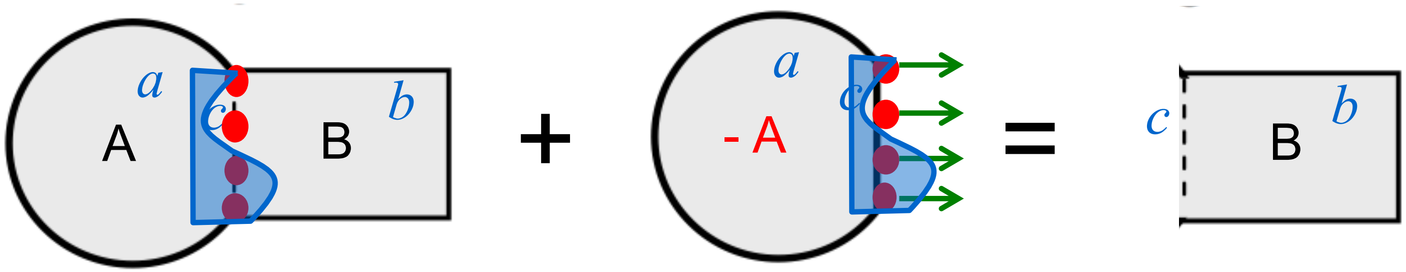


$$\begin{bmatrix} \boldsymbol{Z}^{AB} & 0 & \begin{bmatrix} 0 \\ \boldsymbol{I} \\ 0 \end{bmatrix} \\ 0 & -\boldsymbol{Z}^{A} & \begin{bmatrix} -\boldsymbol{I} \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \boldsymbol{I} & 0 \end{bmatrix} & \begin{bmatrix} -\boldsymbol{I} & 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_a^{AB} \\ \boldsymbol{u}_c^{AB} \\ \boldsymbol{u}_b^{AB} \\ \boldsymbol{u}_a^{A} \\ \boldsymbol{u}_c^{A} \\ \lambda \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_a^{AB} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Frequency Based Decoupling: A Big Family

$$\begin{bmatrix} \boldsymbol{Z}^{AB} & 0 & \begin{bmatrix} 0 \\ \boldsymbol{I} \\ 0 \end{bmatrix} \\ 0 & -\boldsymbol{Z}^{A} & \begin{bmatrix} -\boldsymbol{I} \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \boldsymbol{I} & 0 \end{bmatrix} & \begin{bmatrix} -\boldsymbol{I} & 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_a^{AB} \\ \boldsymbol{u}_c^{AB} \\ \boldsymbol{u}_b^{AB} \\ \boldsymbol{u}_a^{A} \\ \boldsymbol{u}_c^{A} \\ \lambda \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_a^{AB} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We want now to impose compatibility on the interface only for a reduced modal approximation of the displacements on the interface. Assuming we use as representation basis the modes of A (often modeled by FE) on the interface: $\boldsymbol{\Phi}_c^A$. Then



$$\boldsymbol{u}_c^A \simeq \boldsymbol{\Phi}_c^A \boldsymbol{q}^A \rightarrow \boldsymbol{q}^A = \boldsymbol{\Phi}_c^{A^+} \boldsymbol{u}_c^A,$$

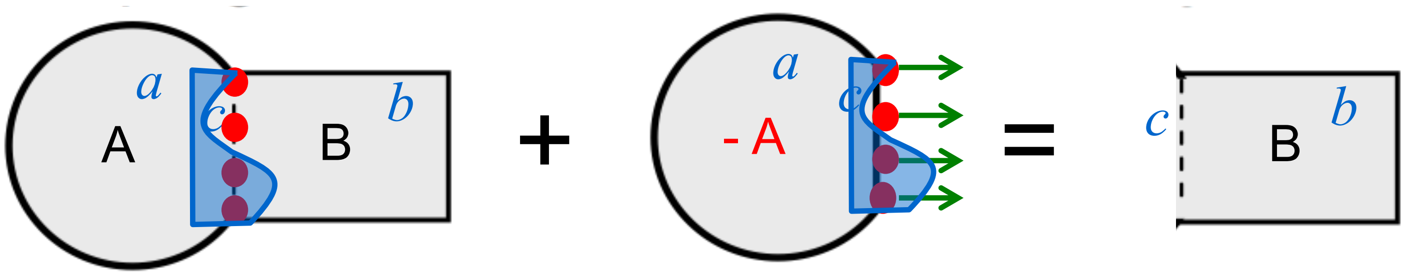
$$\boldsymbol{u}_c^{AB} \simeq \boldsymbol{\Phi}_c^A \boldsymbol{q}^{AB} \rightarrow \boldsymbol{q}^{AB} = \boldsymbol{\Phi}_c^{A^+} \boldsymbol{u}_c,$$

"filtered" compatibility

$$\boldsymbol{q}^{AB} - \boldsymbol{q}^A = \boldsymbol{\Phi}_c^{A^+} \boldsymbol{u}_c - \boldsymbol{\Phi}_c^{A^+} \boldsymbol{u}_c^A = \boldsymbol{0}$$

Frequency Based Decoupling: A Big Family

→ Method 5: ***Weak - interface filtering***

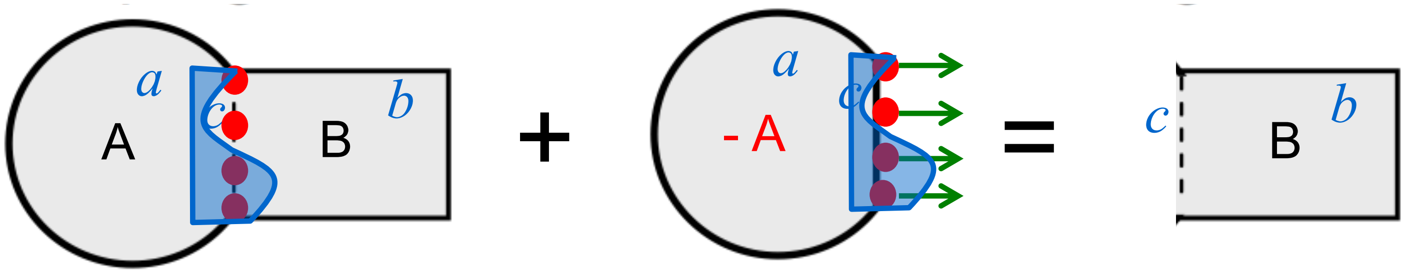


$$\begin{bmatrix} \boldsymbol{Z}^{AB} & 0 & \begin{bmatrix} 0 \\ \boldsymbol{\Phi}_c^{A^{+T}} \\ 0 \end{bmatrix} \\ 0 & -\boldsymbol{Z}^{A} & \begin{bmatrix} -\boldsymbol{\Phi}_c^{A^{+T}} \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \boldsymbol{\Phi}_c^{A^{+}} & 0 \end{bmatrix} & \begin{bmatrix} -\boldsymbol{\Phi}_c^{A^{+}} & 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_a^{AB} \\ \boldsymbol{u}_c^{AB} \\ \boldsymbol{u}_b^{AB} \\ \boldsymbol{u}_a^{A} \\ \boldsymbol{u}_c^{A} \\ \lambda_{n_\Phi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_a^{AB} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

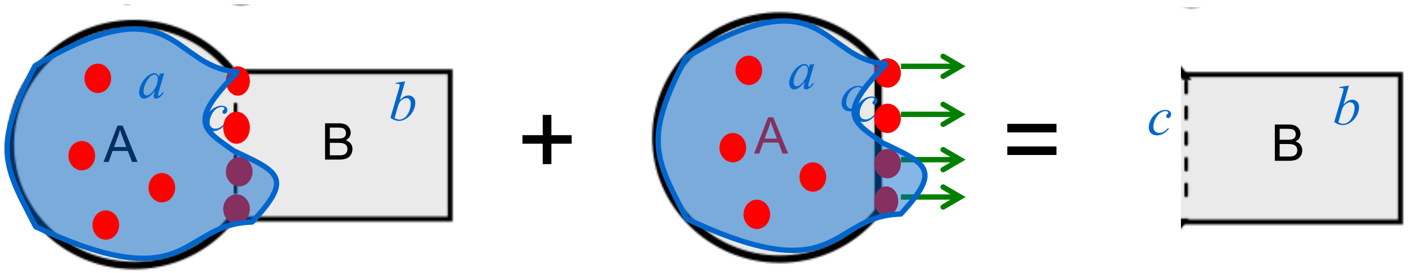
Request compatibility on interface only for modal contributions in space of $\boldsymbol{\Phi}_c^A$

. weak interface compatibility (only n_{Φ} conditions)
. the filtering basis can be any good representation of interface behavior
(here the modes of A, but could be any approximate shape)
. needs enough measured points c on the interface to have a good fit.

Frequency Based Decoupling: A Big Family



Idea: use also the *internal points in A* to fit the modal representation



$$\begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c \end{bmatrix}^{AB} \simeq \begin{bmatrix} \boldsymbol{\Phi}_a^A \\ \boldsymbol{\Phi}_c^A \end{bmatrix} \boldsymbol{q}^{AB} \rightarrow \boldsymbol{q}^{AB} = \begin{bmatrix} \boldsymbol{\Phi}_a^A \\ \boldsymbol{\Phi}_c^A \end{bmatrix}^+ \begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c \end{bmatrix}$$

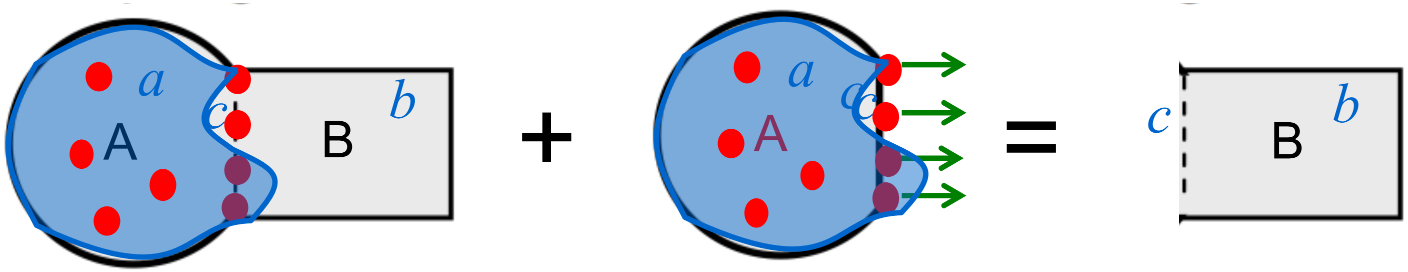
$$\begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c^A \end{bmatrix}^{A} \simeq \begin{bmatrix} \boldsymbol{\Phi}_a^A \\ \boldsymbol{\Phi}_c^A \end{bmatrix} \boldsymbol{q}^{A} \rightarrow \boldsymbol{q}^{A} = \begin{bmatrix} \boldsymbol{\Phi}_a^A \\ \boldsymbol{\Phi}_c^A \end{bmatrix}^+ \begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c^A \end{bmatrix}$$

"filtered" compatibility

$$\boldsymbol{q}^{AB} - \boldsymbol{q}^{A} = \begin{bmatrix} \boldsymbol{\Phi}_a^A \\ \boldsymbol{\Phi}_c^A \end{bmatrix}^+ \begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c \end{bmatrix} - \begin{bmatrix} \boldsymbol{\Phi}_a^A \\ \boldsymbol{\Phi}_c^A \end{bmatrix}^+ \begin{bmatrix} \boldsymbol{u}_a \\ \boldsymbol{u}_c^A \end{bmatrix} = \boldsymbol{0}$$

Frequency Based Decoupling: A Big Family

→ Method 6: ***Weak - A filtering***



$$\begin{bmatrix} \boldsymbol{Z}^{AB} & 0 & \begin{bmatrix} 0 \\ \boldsymbol{\Phi}_{c,a}^{A^{+T}} \\ 0 \end{bmatrix} \\ 0 & -\boldsymbol{Z}^{A} & \begin{bmatrix} -\boldsymbol{\Phi}_{c,a}^{A^{+T}} \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \boldsymbol{\Phi}_{c,a}^{A^{+}} & 0 \end{bmatrix} & \begin{bmatrix} -\boldsymbol{\Phi}_{c,a}^{A^{+}} & 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_a^{AB} \\ \boldsymbol{u}_c^{AB} \\ \boldsymbol{u}_b^{AB} \\ \boldsymbol{u}_a^{A} \\ \boldsymbol{u}_c^{A} \\ \lambda_{n_\Phi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_a^{AB} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Request compatibility on interface & internal dofs of A only for modal contributions $\mathbf{\Phi}_c^A$

. weak interface compatibility (only n_Φ conditions)
. the filtering basis can be any good representation of interface behavi
. use of any (sufficient) info available on A (works even if no interface measurement)

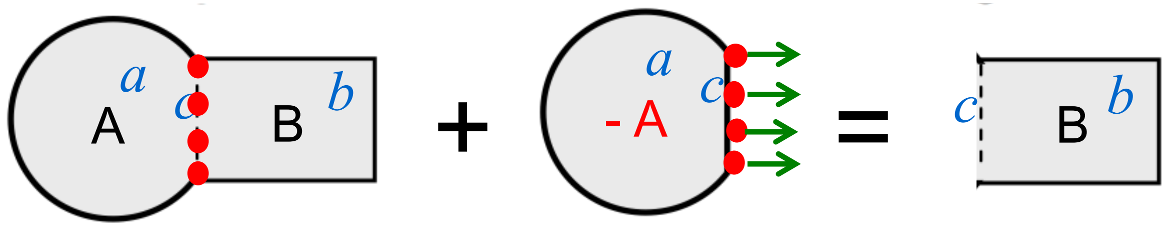
Can be seen as a weakening of the extended interface method (method 2)
This idea was exploited in the Modal Constraints for Fixture & Subsystem (MCFS) → MOD08

[7]

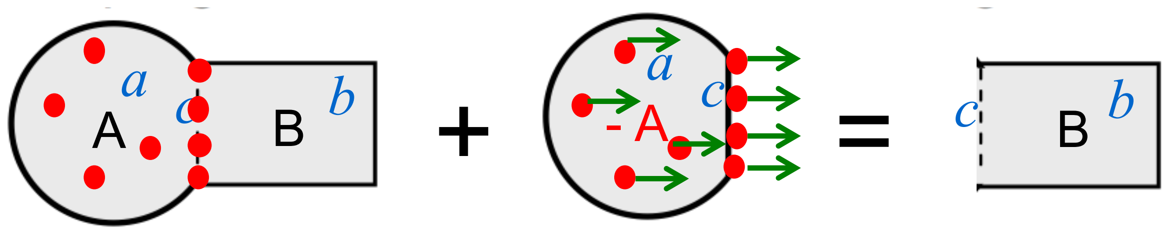
Summary of variants discussed

Collocated (strong comp.)

Meth.1:
Standard

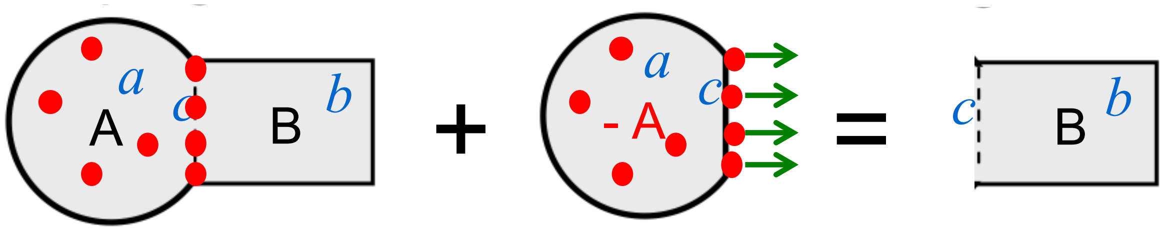


Meth.2:
Extended Interface

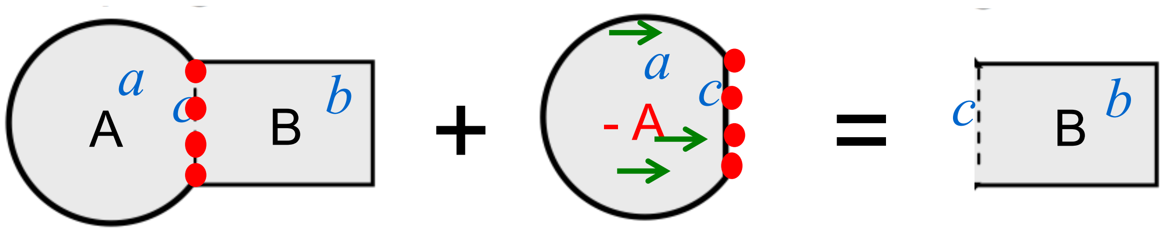


Non-Collocated (usually approx/weak comp.)

Meth.3:
NC overdetermined



Meth.4:
NC internal

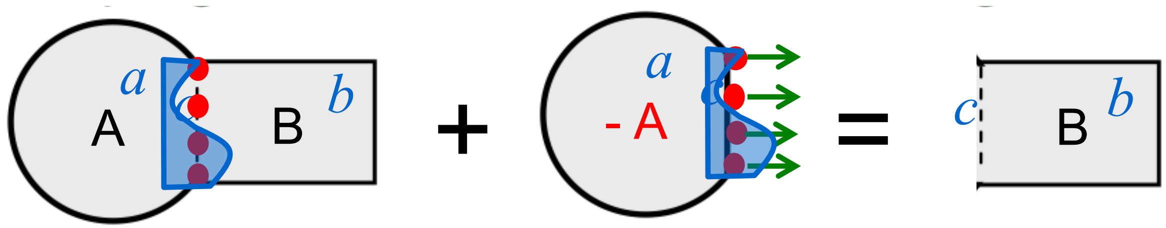


As said, further variants can be built following similar ideas.

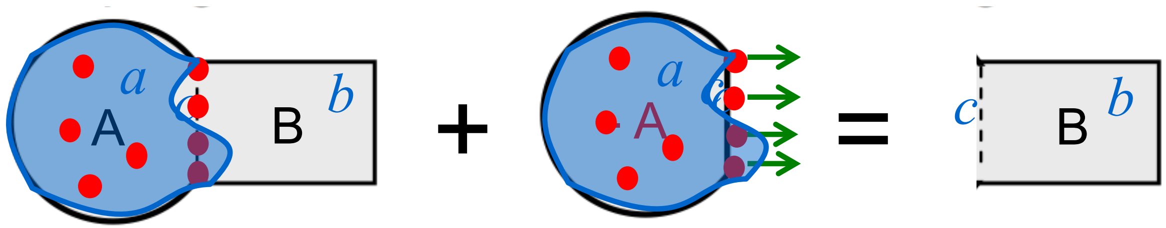
Summary of variants discussed (continued)

<u>Weak Compatibility by filtering</u>

Meth.5:
Weak - Interface Filtering



Meth.6:
Weak – A Filtering



As said, further variants can be built following similar ideas.

Outline

- **Recap of MOD02:**
  Frequency Based Substructuring (FBS)3-field form

- **Frequency Based Substructuring: Basics**

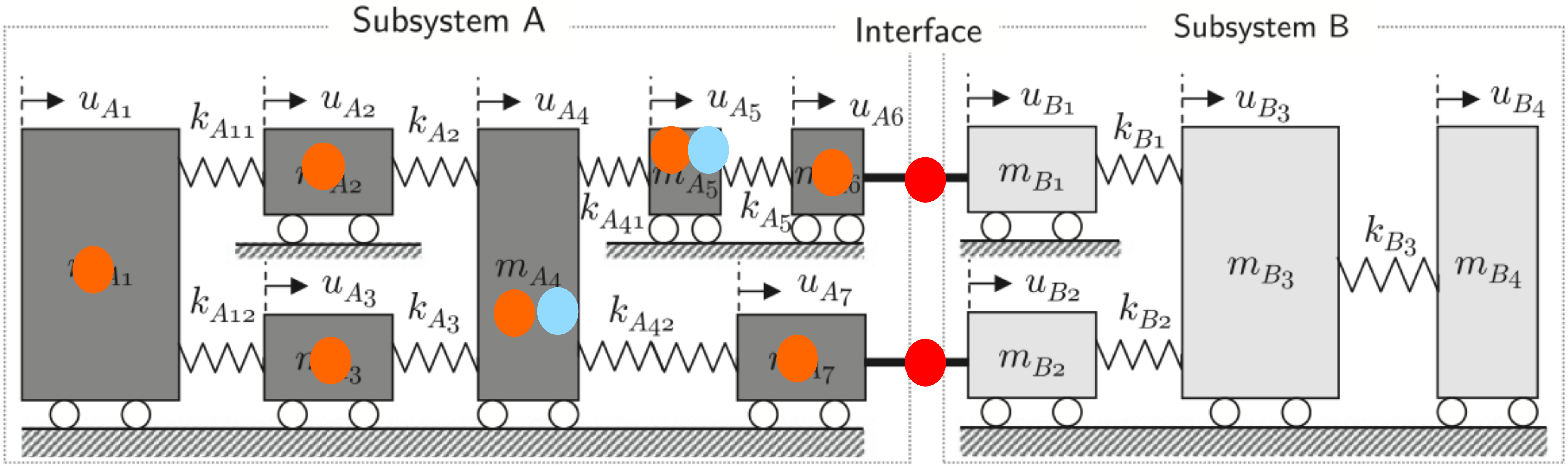
- **Frequency Based Substructuring: A big Family**
  - Method 1 – Standard Decoupling
  - Method 2 – Extended Interface Decoupling
  - Method 4 – Non-Collocated Overdetermined
  - Method 5 – Weak, Interface Filtering
  - Method 6 – Weak, A Filtering
  - Summary of Variants

- **Numerical Example / Experimental Example**

References and bibliography

Numerical example:

data given in [1]



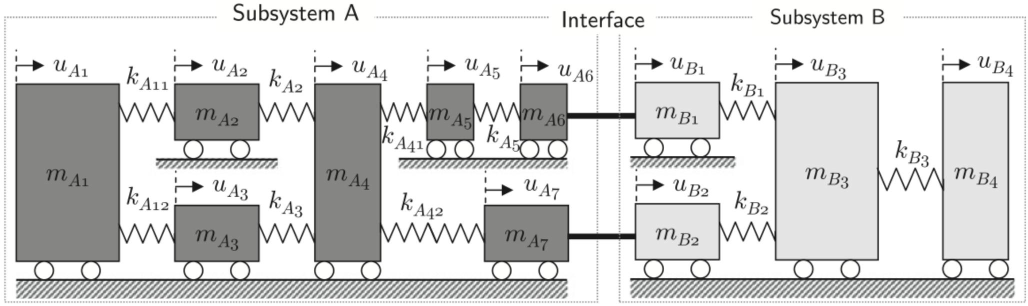
Methods tested:

	dofs in comp.	dofs for forces	No noise	with noise
Meth. 1 Standard	●	●		
Meth. 2 Extended interf.	● ●	● ●		
Meth. 3 NC overdeter.	● ●	●		
Meth. 4 NC internal	●	●		

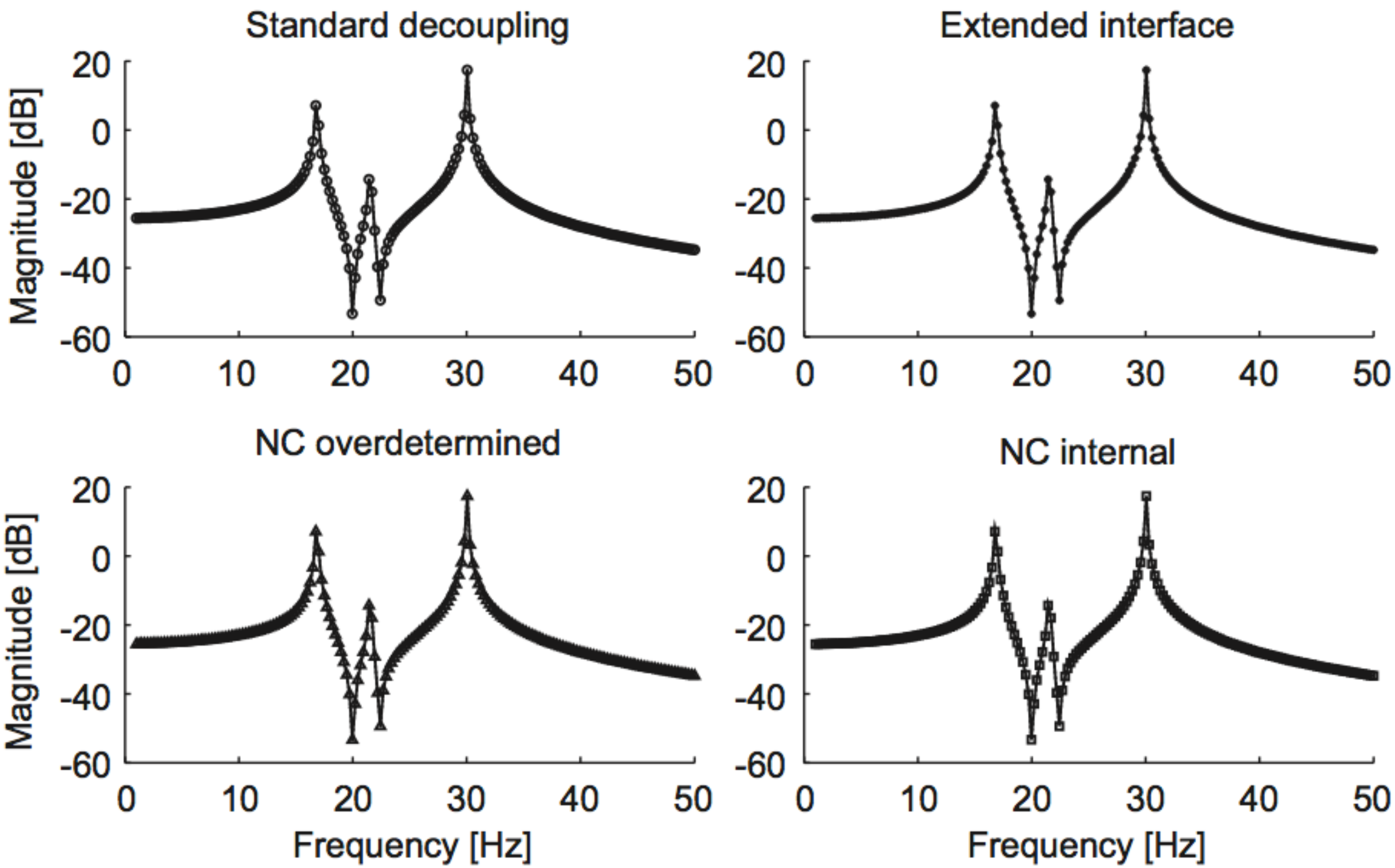
Check consistency

Sensitivity to error

Numerical example:

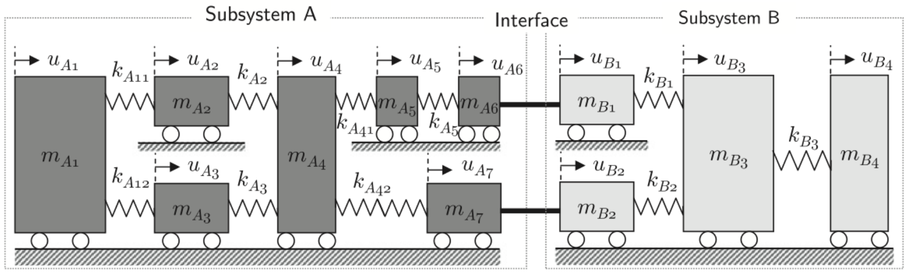


No noise



All 4 methods return the exact solution → consistent

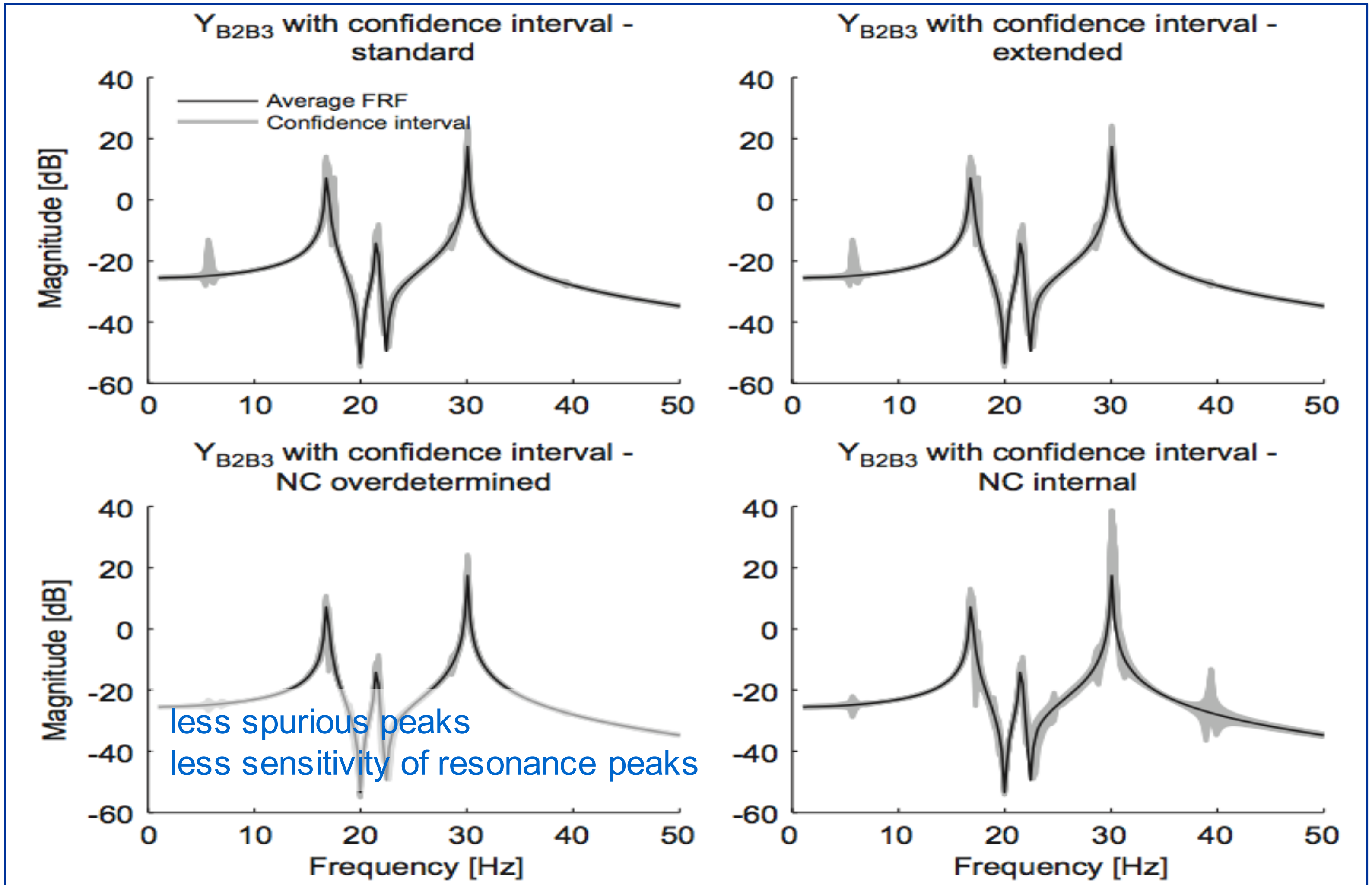
Numerical example:



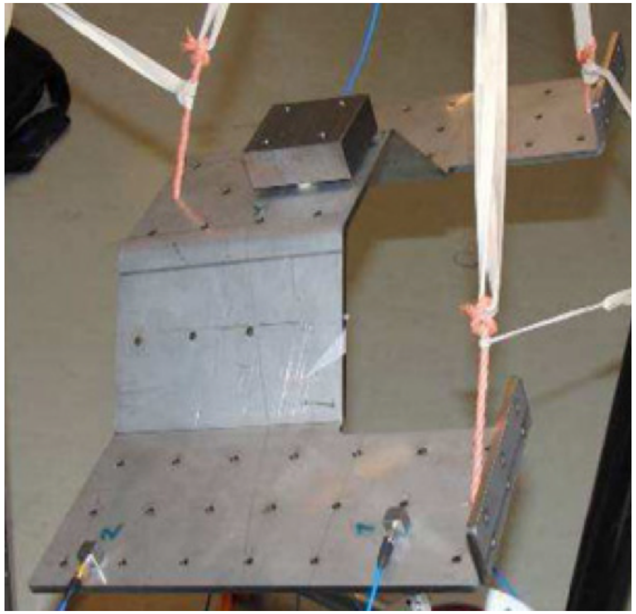
Uncertainty propagation analysis

through Monte-Carlo simulations or formal uncertainty analysis [8]

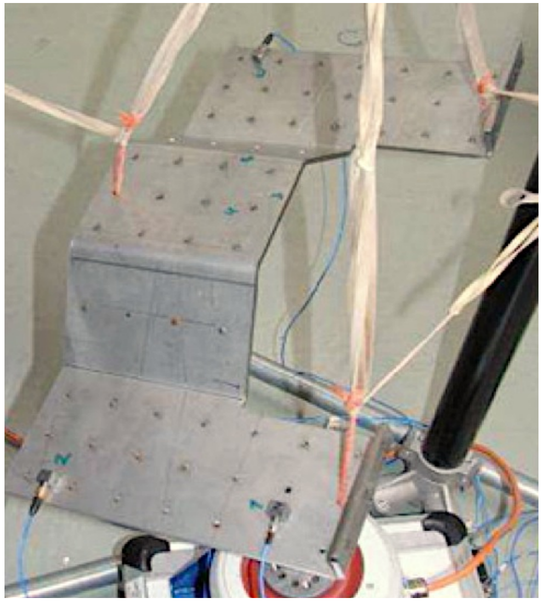
normally distributed random error – amplitude: 95% confidence interval of +/- 2%
phase : 95% confidence interval of +/- 2°



Experimental example



-



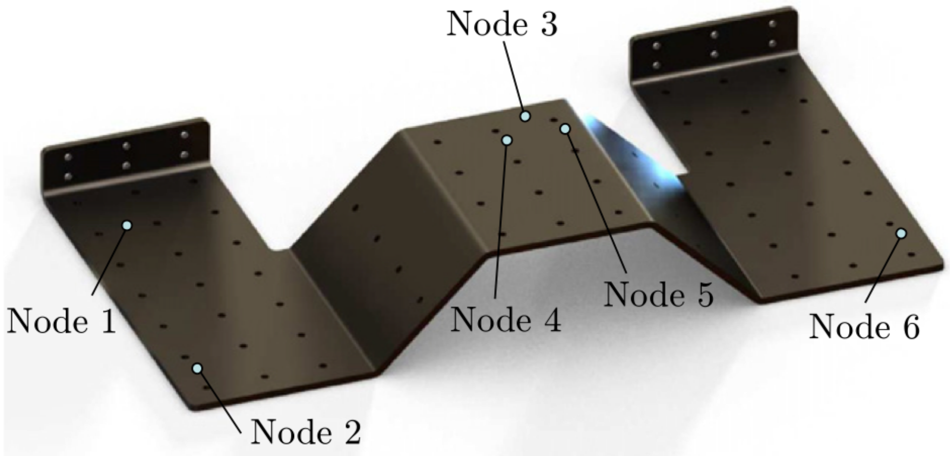
=

rigid mass
(simple FRF)

AB

A

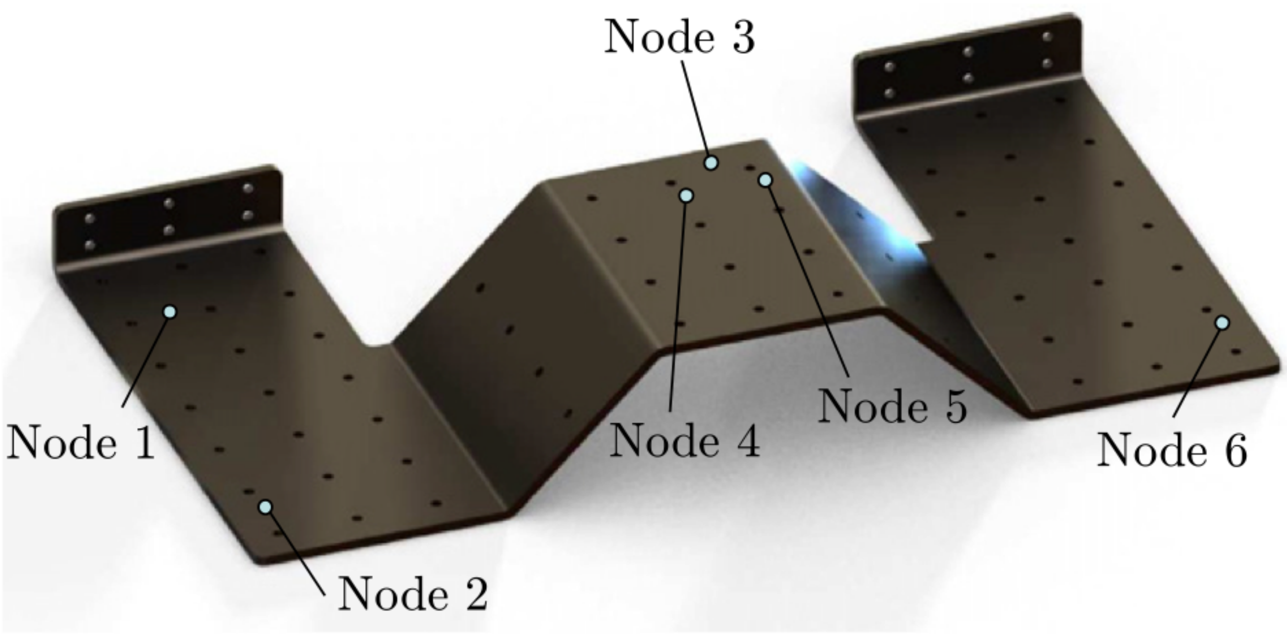
B ?



Experimental example

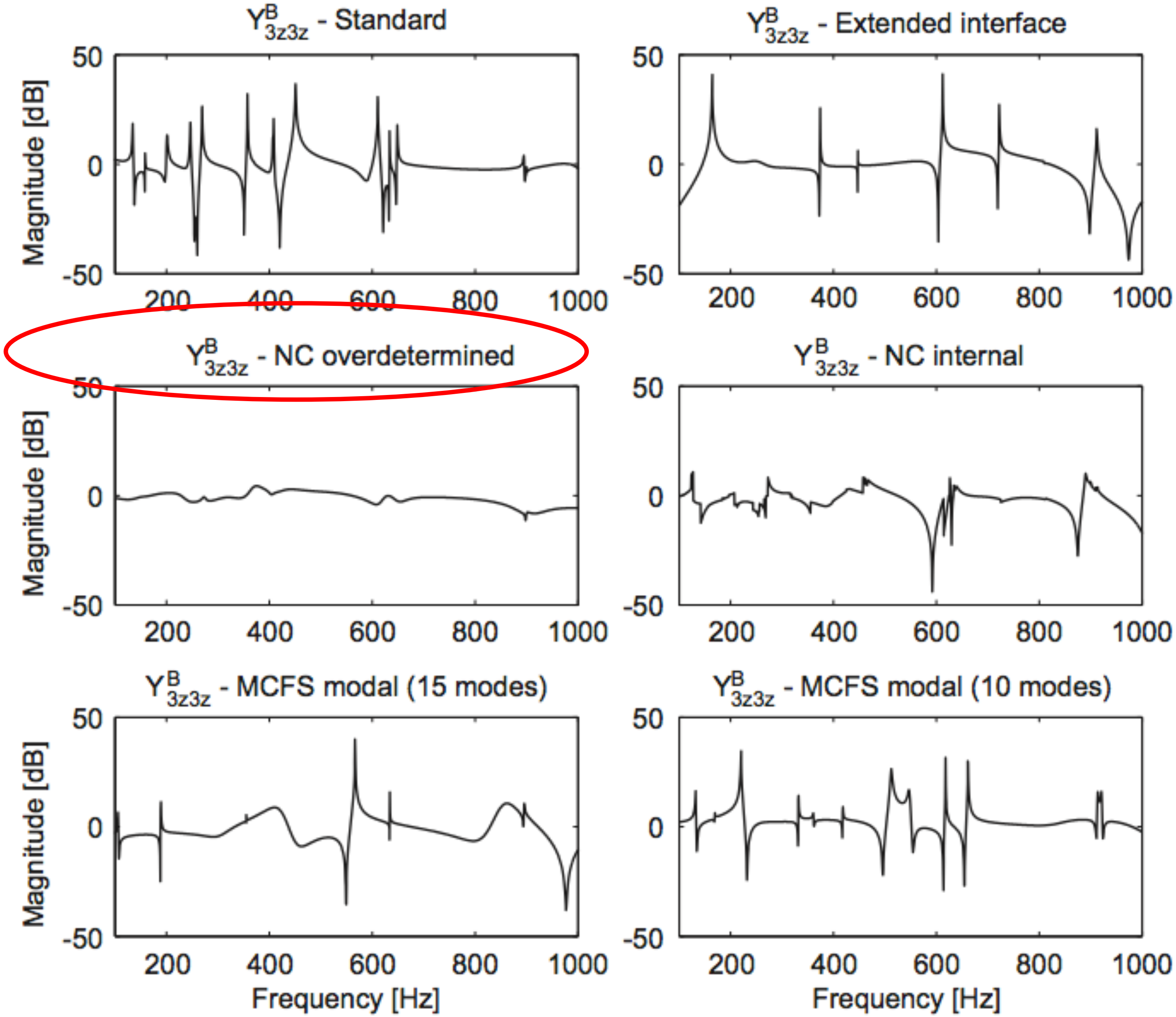
Overview of decoupling variants applied to experimental case study.

Method	Compatibility	Equilibrium
Standard	Nodes 3–5 (9 DoF)	Nodes 3–5 (9 DoF)
Extended interface	Nodes 1–6 (18 DoF)	Nodes 1–6 (18 DoF)
NC overdetermined	Nodes 1–6 (18 DoF)	Nodes 3–5 (9 DoF)
NC internal	Nodes 3–5 (9 DoF)	Nodes 1, 2 and 6 (9 DoF)
MCFS modal	Nodes 1–6 (18 DoF) projected on first 15 modes of A	See compatibility
MCFS modal	Nodes 1–6 (18 DoF) projected on first 10 modes of A	See compatibility



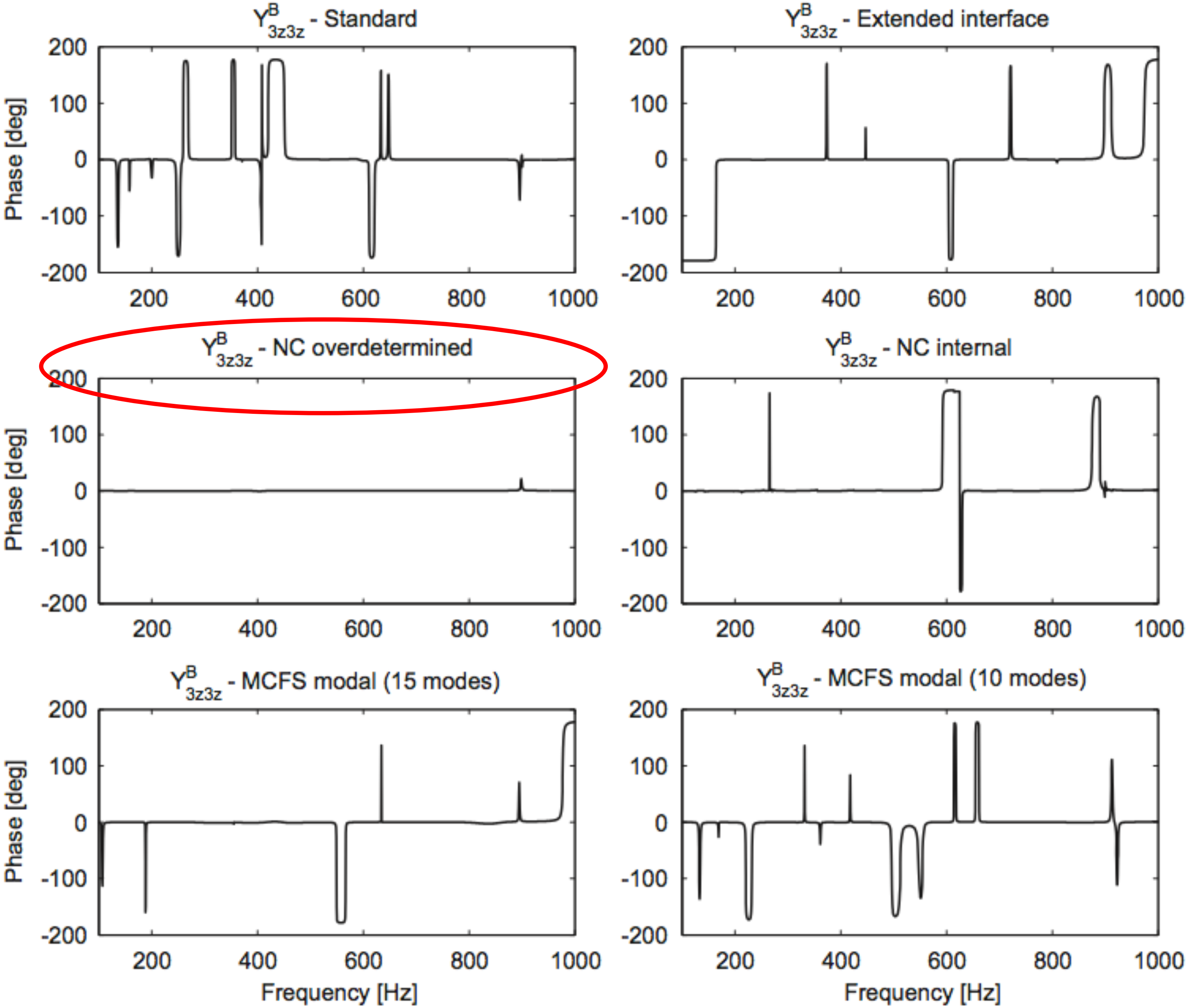
Experimental example

Extract FRF (should be a mass^{-1} line)



Experimental example

Extract FRF (should be a mass^{-1} line)



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